Lecture 9: State Space Models: Part II

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Lecture Objectives:

- Introduction to Dynamic Factor Models (DFM).
- Principal Components
- Factor Augmented VAR (FAVAR)
- Identification of FAVARs
- FAVARs impulse response functions

Secondary Readings:

- “Dynamic Factor Models”, Stock and Watson.
- “Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach”, Bernanke, Boivin and Eliasz.

Intro to DFMs

- As we saw before, VARs provide a flexible yet heavily parameterized characterizations of the interactions of observable time series.
- In many cases, the observables behavior is the result of a small set of unobservable underlying process.
- Example: Real business cycle (RBC) whereby total factor productivity is the driving force behind investment, consumption and output.

Intro to DFMs

- DFMs were developed by Geweke (1977) and can also be represented by a state space representation.
- Hence, the Kalman Filter can be applied. However, since the coefficients are of interest and unknown, we need to estimate them by maximizing likelihood.
- This process with many variables can be cumbersome. Hence, we will show how the literature has evolved over time to estimate DFMs.

Intro to DFMs

- First Generation: time-domain maximum likelihood via the Kalman filter
- Second Generation: nonparametric averaging methods (principal components)
- Third Generation: hybrid principal components and state space methods

DFMs Motivation

- Macroeconometricians face data sets that have large number of series, but the number of observations on each series is relatively short, for example 20 to 40 years of quarterly data.
- Sargent and Sims (1977) showed that two dynamic factors could explain a large fraction of the variance of important U.S. quarterly macroeconomic variables, including output, employment, and prices.

DFMs Motivation

- The premise of a dynamic factor model is that a few latent dynamic factors, f_t, drive the comovements of a high-dimensional vector of time-series variables, X_t, which is also affected by a vector of mean-zero idiosyncratic disturbances, e_t.
- These idiosyncratic disturbances arise from measurement error and from special features that are specific to an individual series (the effect of a Salmonella scare on restaurant employment, for example).
- Finally, the latent factors follow a time series process, which is commonly taken to be a vector autoregression (VAR).

Intro to DFMs

$$
\begin{aligned}
X_t &= \lambda(L) f_t + e_t & (1) \\
f_t &= \Psi(L) f_{t-1} + \eta_t & (2)
\end{aligned}
$$

where there are N series, so that X_t and e_t are both $N \times 1$, there are q factors so that f_t and η_t are both $q \times 1$, L is the lag operator, and the lag polynomial matrices $\lambda(L)$ and $\Psi(L)$ are $N \times q$ and $q \times q$, respectively.

- The i^{th} lag polynomial $\lambda_i(L)$ is called the dynamic factor loading for the i^{th} series, X_{it}.
- And $\lambda_i(L) f_t$ is called the common component of the i^{th} series.

Intro to DFMs

- We assume that all the processes in (1) and (2) are stationary. (Important!)
- The idiosyncratic disturbances are assumed to be uncorrelated with the factor innovations at all leads and lags, $E[e_t \eta'_{t-k}] = 0$ for all k.
- By using DFMs, the forecaster gets the benefit of using all N variables by using only q factors, where q is typically much smaller than N.

Factors Estimation - Principal Components

- The first way to estimate the factors and factor loadings is to use Kalman Filter. We already saw this last lecture. However, if there are a large number of factors, this can be difficult to implement.
- Another way, is to estimate the factors first, and then treat them as data running a VAR on (2).
- One very useful method to estimate factors and their factor loadings is principal components

Factors Estimation - Principal Components

- Lets write (1) as a static model first:

$$X_t = \Lambda F_t + e_t$$

  Where F_t could potentially include lagged factors and Λ is a matrix with all the factor loadings.

- Principal components offers a way to estimate both $\hat{F}_t$ and $\hat{\Lambda}$.

Factors Estimation - Principal Components

- There are many ways in which one can arrive at the principal components estimates.
- Here, we will show how to drive them as a solution to the following least squares problem:

$$\min_{F_1,\ldots,F_T,\Lambda} \frac{1}{NT}\sum_{t=1}^{T}(X_t - \Lambda F_t)'(X_t - \Lambda F_t)$$

subject to $N^{-1}\Lambda'\Lambda = I$

- This is equivalent to maximize:

$$\max_{\Lambda} \Lambda'\hat{\Sigma}_{XX}\Lambda \tag{3}$$

subject to $N^{-1}\Lambda'\Lambda = I$

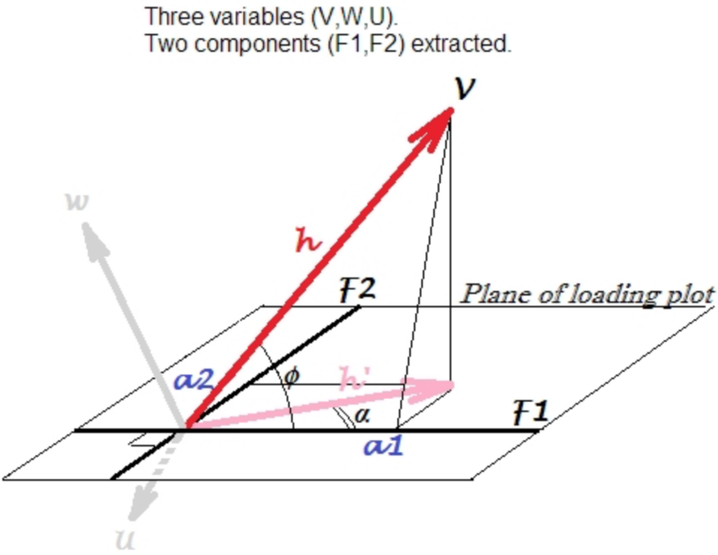
Factors Estimation - Principal Components

- The solution is set the factor loadings matrix equal to the scaled eigenvectors of $\hat{\Sigma}_{XX}$ corresponding to its q largest eigenvalues.
- The principal components $\hat{F}_t$ are going to be given by:

$$\hat{F}_t = N^{-1}\hat{\Lambda}' X_t$$

Factors Estimation - Principal Components

Suppose we have three variables and we compute two principal components:



Factors Estimation - Principal Components

- Suppose X_t has 3 variables: investment, output and consumption.
- And that we are interested in reducing the dimensionality by estimating one factor.
- This can be motivated by the RBC model whereby TFP is the underlying process behind those three variables.
- This is a good example of dimensionality reduction achieved though Principal Components Analysis.

Factors Estimation - Principal Components

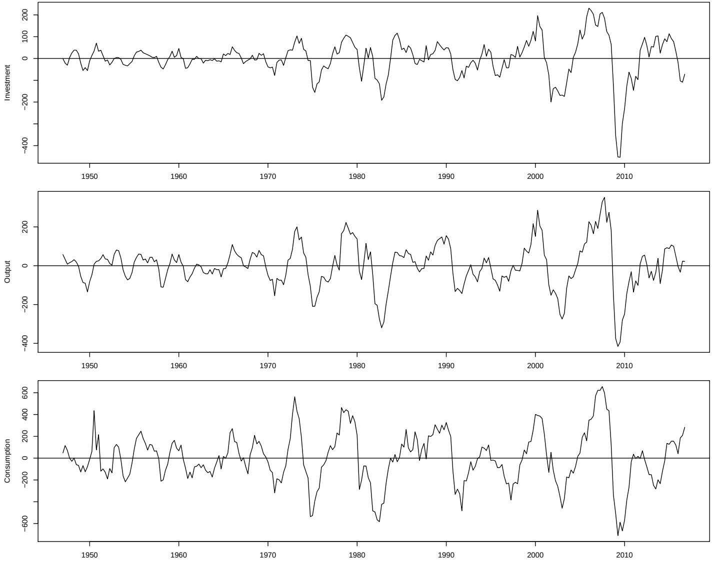
- Let investment be i_t, output y_t and consumption c_t we have:

$$\begin{bmatrix} i_t \\ y_t \\ c_t \end{bmatrix} = \begin{bmatrix} \Lambda_{11} \\ \Lambda_{21} \\ \Lambda_{31} \end{bmatrix} F_{1t} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

- Before estimating the system with only one factor, we can make some analysis with more factors to see whether the data can indeed be mostly explain by only one factor.

Factors Estimation - Principal Components

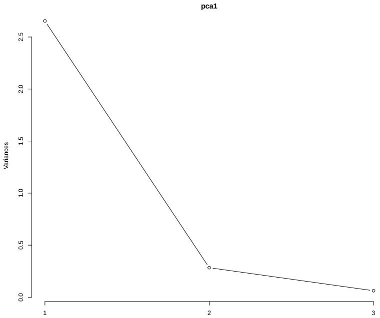
- The business cycle on the data is given by:



Factors Estimation - Principal Components

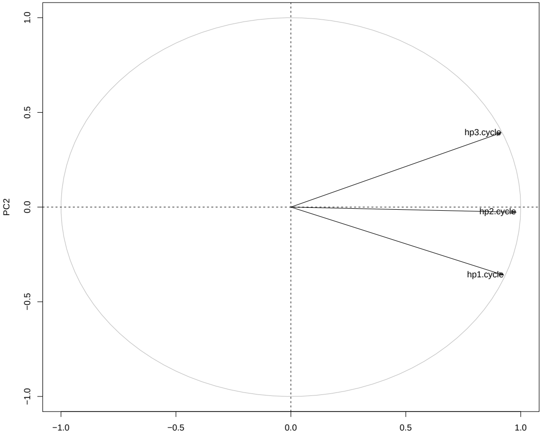
- Lets estimate the model with 3 factors and see how much of the variance is explained by each different factor:

```
Importance of components:
                          PC1     PC2     PC3
Standard deviation     1.6291 0.53268 0.24955
Proportion of Variance 0.8847 0.09458 0.02076
Cumulative Proportion  0.8847 0.97924 1.00000
```



Factors Estimation - Principal Components

- Indeed most of the variation in the data can be explained by the first component



Factors Estimation - Principal Components

- Here is the estimated $\hat{\Lambda}$:

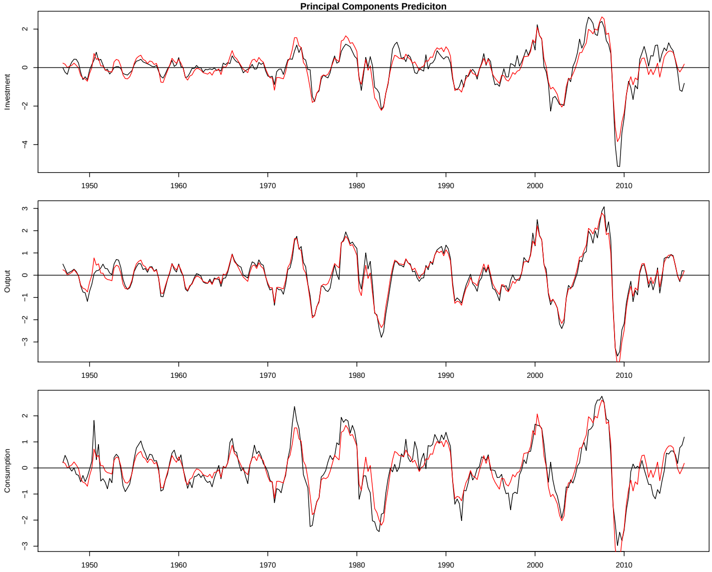
```
> pca1$rotation
                 PC1
hp1.cycle 0.5682794
hp2.cycle 0.6013317
hp3.cycle 0.5616571
```

- And the first five observations of $\hat{F}_1$:

```
> head(pca1$x, n=5)
            PC1
[1,] 0.40923441
[2,] 0.30470630
[3,] 0.00903761
[4,] 0.10913112
[5,] 0.20929588
```

Factors Estimation - Principal Components

- All the predicted series using the estimated factor and loadings:



Back to DFMs

- The second generation of DFMs, uses principal components to estimate the Factors and then treats them as data and estimates the equation (2):

$$\hat{F}_t = \Psi \hat{F}_{t-1} + \eta_t$$

- Which is just a VAR(1) with the estimated factors from PCA.
- The third generation of DFMs, runs in the same way but adds an extra step of re-estimating the Factors using the Kalman Smoother.

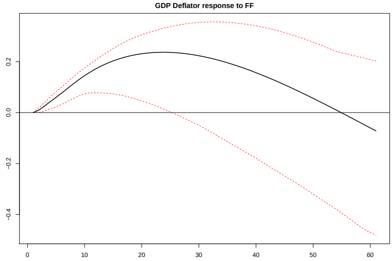
FAVAR

- The FAVAR, proposed by Bernanke, Boivin and Eliasz, applies the DFMs idea in order to improve the identification of shocks in traditional structural VARs.
- The structural VARs had some limitations:
    1. Incorrect identification of shocks: small amount of information used by low-dimensional VARs
    2. Impulse responses can be observed only for the included variables

FAVAR

- Famous example of identification issues with monetary policy shocks: Price Puzzle

- This puzzle refers to the conventional finding in the VAR literature that a contractionary monetary policy shock is followed by an increase in the price level, rather than a decrease as standard economic theory would predict.



FAVAR

- A common explanation for the price puzzle was for the lack of information in the VAR system that might be in the information set of the central bank.
- As BBE pointed out “If the Fed systematically tightens policy in anticipation of future inflation, and if these signals of future inflation are not adequately captured by the data series in the VAR, then what appears to the VAR to be a policy shock may in fact be a response of the central bank to new information about inflation.”

FAVAR

- The question then is: Is it possible to include a rich information set in a VAR without giving up too many degrees of freedom?
- BBE showed that the answer is positive. It can be done with a combination of VAR with factor analysis.
- With the developments of Stock and Watson (2002) DFMs that showed that a large number of macroeconomic series could be well explained by a small number of factors, BBE proposed to include such factors in a VAR, thus calling it factor augmented VAR (FAVAR).

FAVAR

- Let Y_t be an $M \times 1$ vector of observable variables that have strong effects on the rest of the economy.
- Let F_t be an $K \times 1$ vector of unobserved factors. We can think of business activity as an example of an unobserved factor.
- The FAVAR model is given by:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t$$

FAVAR

- But F_t are not observable. However, we assume that there are N variables X_t that are correlated with both Y_t and F_t:

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t$$

  where Λ^f is an $N \times K$ matrix and Λ^y is $N \times M$. They are once again the loading factors.

- There are two ways of estimating the FAVAR. The first one is a two-step principal component. The second one is Bayesian estimation. It turns out that in this case, the Bayesian estimation added complexity does not yield different results. So we will discuss here the 2 step procedure only.

FAVAR

- The first step is to do a PCA in order to estimate the factors. In this step we do not make use of the fact that we observe Y_t. In the end, the principal components still recover consistently the space covered by Y_t and F_t as along as we include enough factors.
- Then, we estimate $\hat{F}_t$ as to be the factors that span X_t that is not spanned by Y_t
- Then the second step is to estimate the VAR using $\hat{F}_t, Y_t$.

FAVAR - Application to Monetary Policy

- Assume Y_t is the Federal Funds Rate.
- Then, in order to identify the monetary policy shock, we assume, like in the structural VAR literature, contemporaneous restrictions.
- In particular, we assume that the federal funds rate affects contemporaneously what we will call "fast moving" variables such as stock prices for instance and that will affect with a lag "slow moving" variables such as output.

FAVAR - Application to Monetary Policy

- After, we estimate the principal components, we need to clean them from the effect of interest rate. However, given the identification assumption, that slow variables are not affected by interest rates contemporaneously, we need to take into account that part of the principal components are not affected by interest rates.

- Hence, we clean them in the following way:
  1. We estimate the factors associated with the slow moving variables $\hat{F}^s$
  2. We estimate the correlations of the components with the interest rate:

  $$\hat{C}_t = b_1 \hat{F}^s + b_2 Y_t + e_t$$

  3. Finally, we clean the factors:

  $$\hat{F}_t = \hat{C}_t - b_2 Y_t$$

FAVAR - Application to Monetary Policy

- Finally we can estimate the VAR with the estimated factors using the recursive identification whereby interest rate is ordered last as now it does not affect any factor contemporaneously (we have cleaned the effect on fast moving variables in the last step).
- The impulse responses of X_t can be recovered by the responses of the factors and interest rates to a shock in interest rate using the mapping between factors and observables:

$$\frac{\partial X_t}{\partial \varepsilon_{Y_t}} = \Lambda^f \frac{\partial \hat{F}_t}{\partial \varepsilon_{Y_t}} + \Lambda^y \frac{\partial Y_t}{\partial \varepsilon_{Y_t}}$$

FAVAR - Application to Monetary Policy

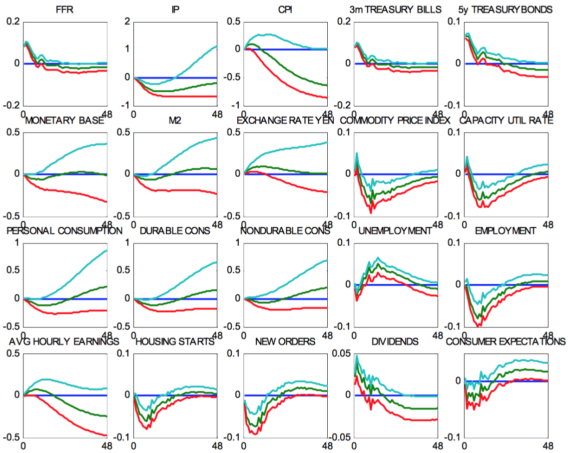


Figure 1. Impulse responses generated from FAVAR with 3 factors and FFR estimated by principal components with 2 step bootstrap.