Lecture 5: Non-stationary models: modeling trends and cycles - Part I

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Lecture Objectives:

- Learn the canonical random walk model and its variations.
- How to remove a trend? Detrending vs Differencing the data.
- Dickey-Fuller tests for stationarity.
- Seasonality and structural breaks.
- Introduction to the HP filter.

Secondary Readings:

- Chapter 4, Applied Econometric Time Series, Enders, Walter, Fourth Edition
- Chapters 15, 16, 17 and 18, Time Series Analysis, Hamilton, James, first edition

Examples of non-stationary series

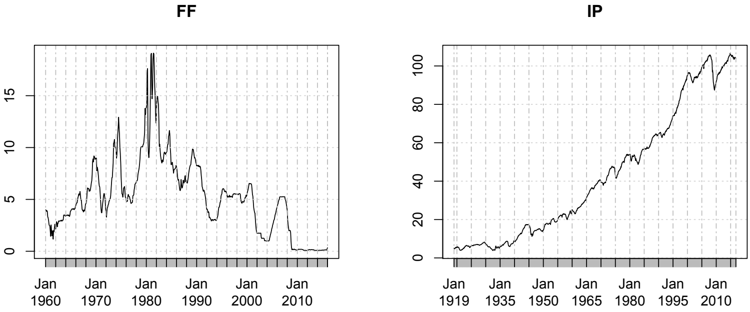


Figure: Federal funds rate and Industrial Production Index time series

Random Walk

- The non-stationary canonical model is the random walk model:

$$y_t = y_{t-1} + \varepsilon_t \quad \text{or } (\Delta y_t = \varepsilon_t) \tag{1}$$

- Note that the in first difference Δy_t the model is stationary.
- Given an initial condition y_0 we can find the solution to (1):

$$y_t = y_0 + \sum_{i=1}^{t} \varepsilon_i \tag{2}$$

Random Walk

- Its mean is:

$$E(y_t) = y_0 \tag{3}$$

- However the variance and autocovariance are time dependent

$$\begin{aligned} var(y_t) &= var(\varepsilon_t + ... + \varepsilon_1) = t\sigma^2 & (4) \\ cov(y_t, y_{t-j}) &= (t-j)\sigma^2 & (5) \end{aligned}$$

- We can also find the ACF:

$$\rho_j = \left[\frac{(t-j)}{t}\right]^{0.5} \tag{6}$$

Random Walk

- Hence, the random walk model is clearly non-stationary.
- Moreover, its ACF has a very specific behaviour.
- The initial lags autocorrelation is close to one and then it decays very slowly.
- Thus, it is not possible to use the autocorrelation function to distinguish between a unit root process and a stationary process with an autoregressive coefficient that is close to unity.

The Random Walk Plus Drift Model

- Now lets add a drift to the canonical model

$$y_t = y_{t-1} + a_0 + \varepsilon_t \tag{7}$$

- Given an initial condition y_0 we get:

$$y_t = y_0 + a_0 t + \sum_{i=1}^{t} \varepsilon_i \tag{8}$$

- y_t is governed by two non-stationary components: a deterministic trend $a_0 t$ and a stochastic trend $\sum_{i=1}^{t} \varepsilon_i$.
- Again, if we take the first difference, we get a series that is stationary (check!)

General Trend Model

$$y_t = y_0 + a_0 t + \sum_{i=1}^{t} \varepsilon_i + A(L)\eta_t \tag{9}$$

$y_t =$ **Deterministic Trend + Stochastic Trend + Stationary**

- This is a very general model of non-stationary time series.
- We can make use of this model to identify if the source of non-stationarity comes from trend

Which Trend model?

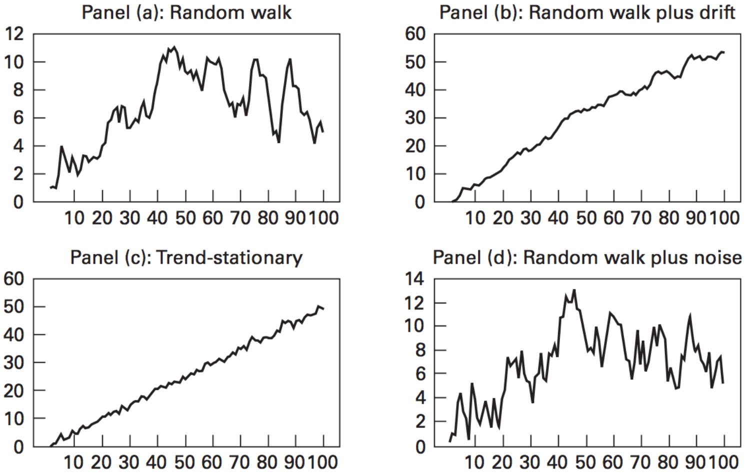


FIGURE 4.2 Four Series With Trends

Figure: Various Trend Models

How to make a non-stationary series stationary?

- It depends on what is the source of non-stationarity.
- In the previous case (9), it is both a deterministic trend and stochastic trend.
- However, there are other sources of non-stationarity such as:
  1. Seasonality
  2. Structural Breaks
- We will deal with the latter two later. Now lets get back to trends.

How to remove Trend?

- There are two ways to make a trend series stationary.
  1. Differencing
  2. Detrending
- One needs to check which method is appropriate depending on the type of type of trend model that is describing the series.

How to remove Trend?

- Suppose the model have a stationary component:

$$y_t = y_0 + a_1 t + \varepsilon_t$$

- The first difference in this case is not well-behaved:

$$\Delta y_t = a_1 + \varepsilon_t - \varepsilon_{t-1}$$

- Δy_t is not invertible because of the unit coefficient in ε_{t-1}
- In this case we have to detrend. If we run a regression $y_t = y_0 + a_1 t + \varepsilon_t$, we can subtract the OLS predicted $\hat{y}_t$ from y_t to recover a stationary series $\hat{\varepsilon}_t$

How to remove Trend?

- If the model has a unit root (also called unit root model) $\Rightarrow$ We should difference **DS**.
- If the model is trend stationary $\Rightarrow$ We should detrend **TS**.

  Example: Below we conclude that REAL GNP is **DS**

Table 4.1 Selected Autocorrelations From Nelson and Plosser

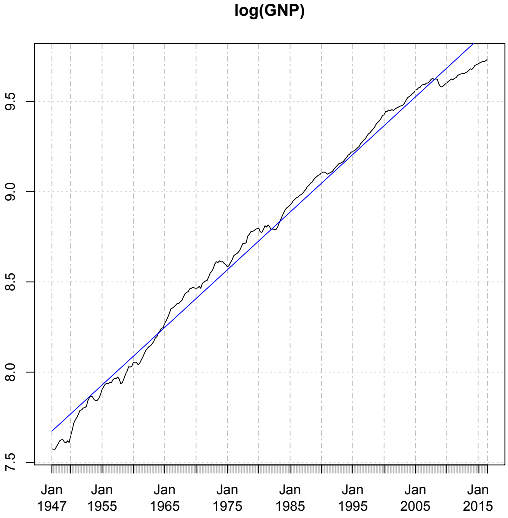
	ρ_1	ρ_2	$r(1)$	$r(2)$	$d(1)$	$d(2)$
Real GNP	0.95	0.90	0.34	0.04	0.87	0.66
Nominal GNP	0.95	0.89	0.44	0.08	0.93	0.79
Industrial production	0.97	0.94	0.03	−0.11	0.84	0.67
Unemployment rate	0.75	0.47	0.09	−0.29	0.75	0.46

Notes:
[1] Full details of the correlogram can be obtained from Nelson and Plosser (1982), who report the first six sample autocorrelations.
[2] ρ_i, $r(i)$, and $d(i)$ refer to the ith-order autocorrelation coefficient for each series, for the first difference of the series, and for the detrended values of the series, respectively.

Figure: Trend VS Detrend

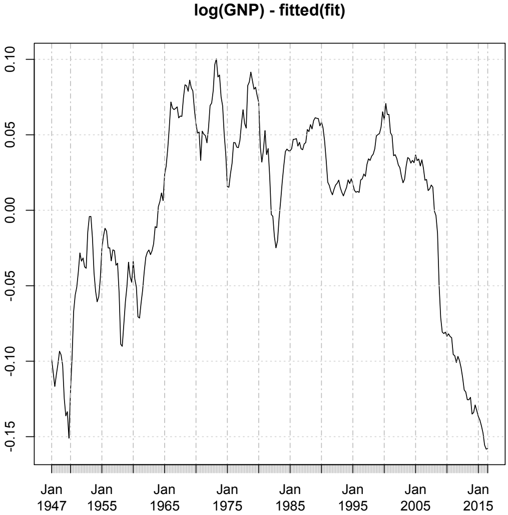
How to remove Trend?

- Lets look graphically. GNP does not seem to be trend stationary:



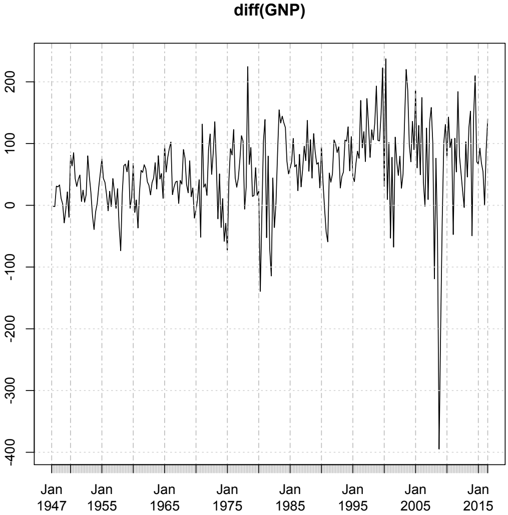
How to remove Trend?

- Indeed the detrended series look far from stationary:



How to remove Trend?

- However the first difference look “fairly” stationary:



Dickey-Fuller Tests for Stationarity

- It is hard to simply use plots or ACF to determine if a series is stationary or not. Two different statisticians might reach different conclusions looking at the previous plot.
- Dickey and Fuller developed the following tests:

$$\begin{aligned}
\Delta y_t &= \gamma y_{t-1} + \varepsilon_t && (10)\\
\Delta y_t &= a_0 + \gamma y_{t-1} + \varepsilon_t && (11)\\
\Delta y_t &= a_0 + \gamma y_{t-1} + a_2 t + \varepsilon_t && (12)
\end{aligned}$$

- The test involves in testing $\gamma = 0$.
    1. The first equation tests for a stochastic trend.
    2. The second tests for a stochastic trend plus deterministic trend
    3. The third one tests for stationary trend model.

Dickey-Fuller Tests for Stationarity

- However, we cannot use classical inference on γ because the series with $\gamma = 0$ are non-stationary and hence sample statistics are not ergodic.
- Dickey and Fuller then used Monte Carlo methods to simulate the distribution of γ in such cases so that we can use it to test for $\gamma = 0$
- An additional challenge is that the distribution changes with the specific parametric form. Hence all (10), (11), (12) will yield different critical values.

  Moreover, we assume that ε_t is stationary!

Dickey-Fuller Tests for Stationarity

- To deal with that, the Dickey-Fuller test can be augmented forming the ADF test:

$$
\begin{aligned}
\Delta y_t &= \gamma y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t && (13) \\
\Delta y_t &= a_0 + \gamma y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t && (14) \\
\Delta y_t &= a_0 + \gamma y_{t-1} + a_2 t + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \varepsilon_t && (15)
\end{aligned}
$$

- The advantage is that with the appropriate lag p, ε_t will be stationary.

Dickey-Fuller Tests for Stationarity

- Lag-selection procedure:
  1. Begin with large enough lag p
  2. If β_p is insignificant using a t-statistic, we remove it and redo it with lag $p-1$
  3. Repeat until the last lag is significant.
- It can be shown that such procedure will select the true model if the first chosen lag p includes the true model
- Moreover, although t-statistics are not valid for γ they are valid for β

Dickey-Fuller Tests for Stationarity

- Example: back to the GNP example.

```
    Augmented Dickey-Fuller Test

data:  detrend_GNP
Dickey-Fuller = -1.7312, Lag order = 2, p-value = 0.6893
alternative hypothesis: stationary

> adf.test(diff_GNP, k=2)

    Augmented Dickey-Fuller Test

data:  diff_GNP
Dickey-Fuller = -7.6484, Lag order = 2, p-value = 0.01
alternative hypothesis: stationary
```

Figure: ADF using the adf.test in R - This function always detrend (It gives the test more power). So we cannot know apriori if the series is stationary or trend stationary. In this case, given that the difference is stationary we know it is a DS process

Dickey-Fuller Tests for Stationarity

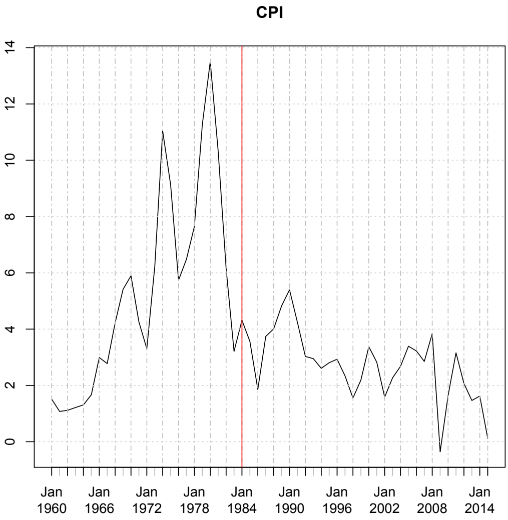
- One concern with the ADF is the lack of power in its test.
- It is very hard to distinguish between a stationary process with a root that is very close to unity and a unit-root process.
- Some methods were developed to improve the power of the test. Elliott, Rothenberg, and Stock (ERS, 1996) suggest that we first estimate the trend components, detrend the series and then apply the ADF.
- This is what the adf.test does in R.

Seasonality

- Sometimes, just taking differences or detrending is not enough to ensure stationarity.
- If there is a strong seasonal component, then our series will not be stationary after doing the usual data transformations.
- In this case, visual inspection of the series can help together with its ACF.
- One way to formaliy test for seasonality is to use the HEGY test (Hylleberg, S., Engle, R., Granger, C. and Yoo, B. (1990))

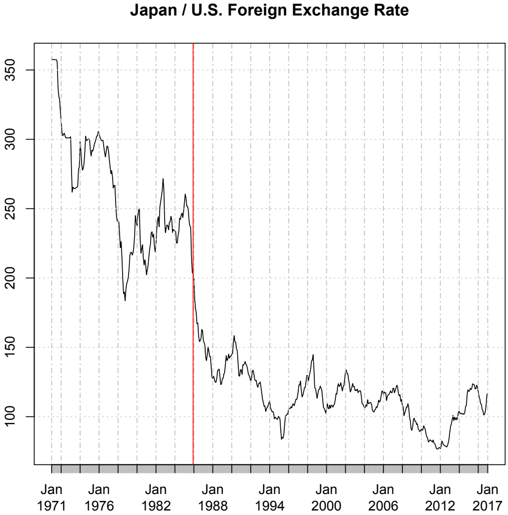
Structural Change

- Another reason why some series are non-stationary is the existence of structural breaks in the series.



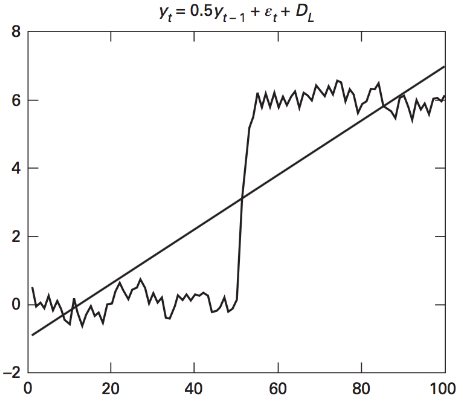
Structural Change

- Another example: US Dollar / Yen. Plaza Accord in 1985



Structural Change

- When there are structural breaks, the various Dickey-Fuller test statistics are biased toward the nonrejection of a unit root.
- Perron (1989) showed that the ADF is indeed biased in the presence of structural breaks



Structural Change

- When it is obvious in the series, one can split the sample and use the ADF test in each sub-sample series.
- However, we will loose degrees of freedom.
- Perron proposed a single test using the full sample.

Hodrick-Prescott (HP) Filter

- The HP (1997) filter is a very useful decomposition of a series into a trend and stationary component.
- The objective is to decompose y_t into a trend μ_t plus a stationary $y_t - \mu_t$. Consider the sum of squares:

$$\frac{1}{T}\sum_{t=1}^{T}(y_t - \mu_t)^2 + \frac{\lambda}{T}\sum_{t=2}^{T-1}[(\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1})]^2 \qquad (16)$$

where λ is a constant and T is number of observations.

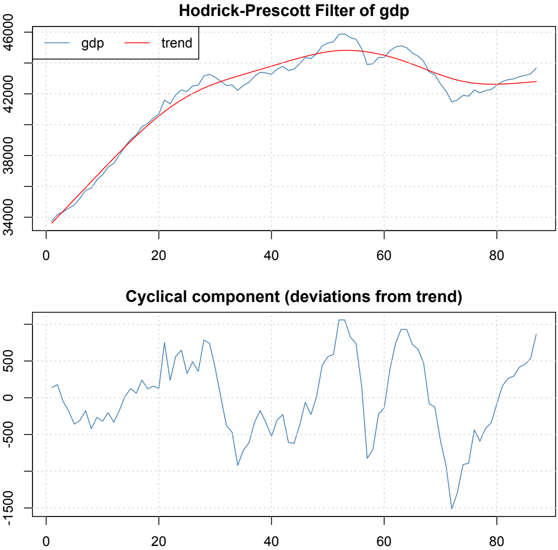
Hodrick-Prescott (HP) Filter

- The idea is to, given λ choose μ_t to minimize (16).
- If $\lambda \to 0$, $\mu_t = y_t$, and if $\lambda \to \infty$, $\mu_t = a_0 t$, just a linear trend.
- The HP filter is vastly applied in the Business Cycle literature
- Typically, these are the values of λ used:

Monthly data	$\lambda = 129,600$
Quarterly data	$\lambda = 1,600$
Annual data	$\lambda = 6.25$

Hodrick-Prescott (HP) Filter

- Example of the HP filter to the Portuguese GDP:



Summary

- Series can contain trends with stochastic and deterministic components
- Differencing can remove a stochastic trend, and detrending can eliminate a deterministic trend.
- However, it is inappropriate to difference a trend-stationary series and to detrend a series containing a stochastic trend.
- We can use the ADF test for stationarity
- Most macroeconomic series contain a stochastic trend. Often time they are I(1) and sometimes I(2)
- In this case it is hard to estimate the stochastic trend. HP filter provides one way to do it.

Questions to think about

- What is a stochastic trend?
- Under what circumstances should we detrend or difference a series?
- Why can't the ADF use standard tests to test for stationarity?
- What are some of the sources for non-stationarity other than trends?
- Why is the HP filter useful for macroeconomics data?